ESDI Technical Note on Bounding EMI Spectrum for Digital Pulse Train
Last Updated Feb 2023

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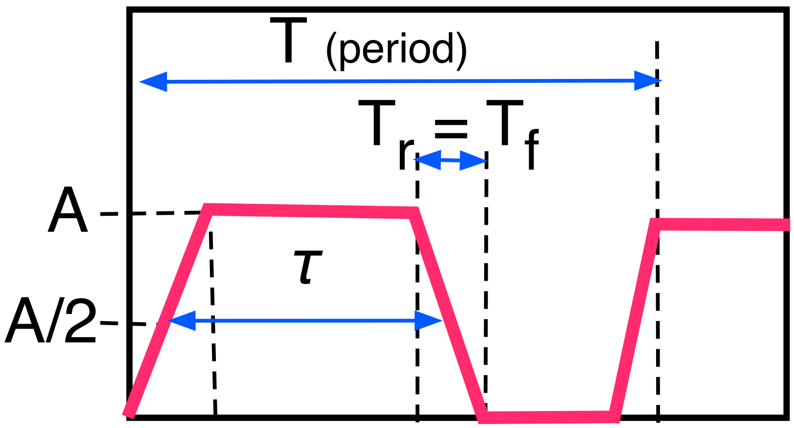
Overview:

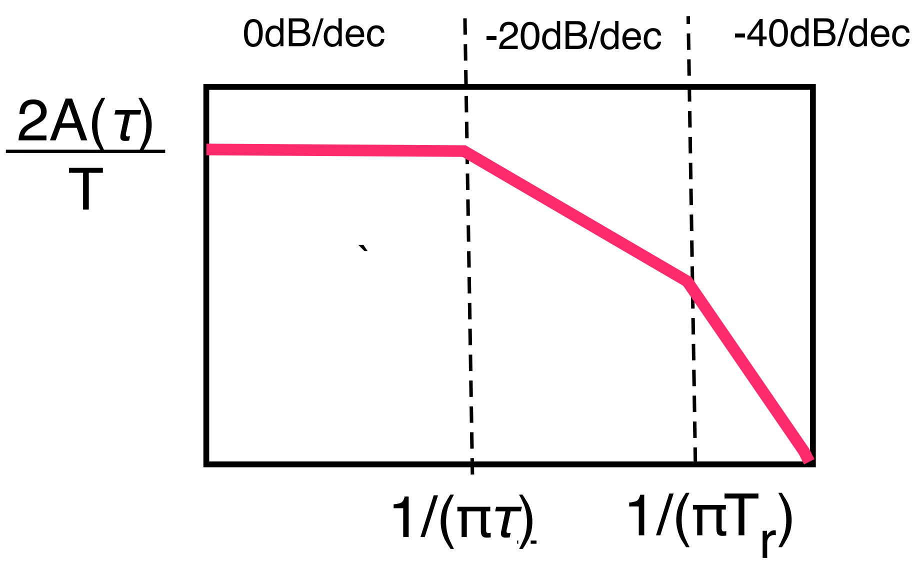
The following is summarized from the references given below.

A Digital Trapezoidal Waveform such as a PWM function is considered with the following simplifications:

1. It is continuous (steady state).
2. T_r and T_f (rise and fall times are equal).
3. The pulse repetition frequency (period) is constant.



This type of waveform creates a comb spectrum… A continuous series of frequency spikes at a spacing of F = 1/T (pulse repetition frequency) following a sin(x)/x envelope pattern and with bounding amplitude asymptotes as follows:



There are three distinct zones in this approximation of the spectrum…

1. Between $F = 0$ and $F = 1/(\pi\tau)$, the amplitude is constant.
2. Between $F = 1/(\pi\tau)$ and $F = 1/(\pi T_r)$, the max amplitude falls off at -20dB/decade (1/F)
3. Above $F = 1/(\pi T_r)$, the max amplitude falls off at -40 dB/decade ($1/F^2$).

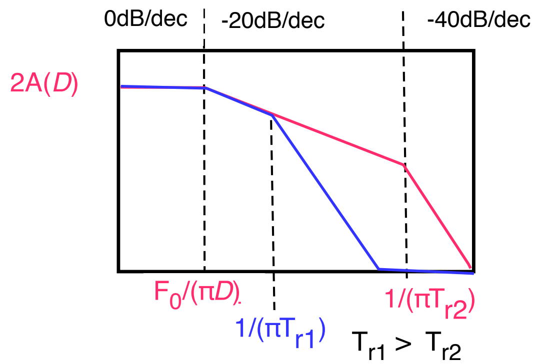
   Pulse Frequency $F_0 = 1/T$, and
   Duty Cycle $D = \tau/T$

The first breakpoint F_1 is at $1/(\pi\tau) = F_0/(\pi D)$. ($D > 0$)
The second breakpoint F_2 is at $1/(\pi T_r)$. ($T_r > 0$)
The peak Amplitude is $2A(\tau)/T = 2A(\tau) F_0 = 2AD$. The maximum amplitude where $(\tau)/T \rightarrow 1$ … D (duty cycle) → 100%, is 2A.

Effect of Pulse Amplitude: The frequency domain amplitude is proportional to the time domain pulse train amplitude… A lower amplitude pulse train with have a lower amplitude frequency spectrum.

Effect of Pulse Period: The spacing of frequency spikes in the spectrum is F_0 (with harmonics at multiples of F_0. For a given duty cycle, the maximum amplitude increases with F_0. Thus, a lower pulse frequency will have a lower amplitude frequency spectrum.

Effect of Pulse Rise Time: As rise time gets very short, F_2 becomes large. F_2 is where the roll off goes as $1/F^2$ (-40dB/decade) and thus falls off much more quickly. This means spectra for pulse trains with longer rise times will fall off at much lower frequency than those with fast rise times. Note that rise time does not affect the overall amplitude…



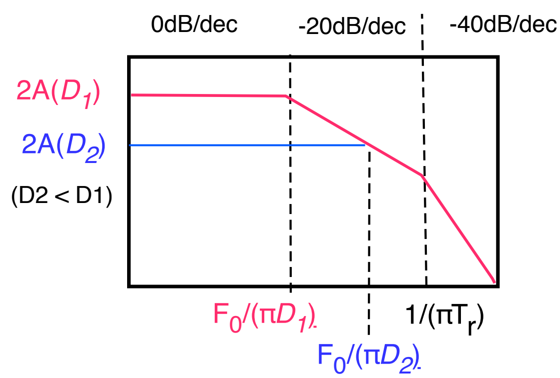
Fast (small) rise times means much more high frequency content and to higher maximum frequencies.

Since spectrum levels fall of very fast above $F_2 = 1/(\pi T_r)$ (i.e., 40 dB/decade), it is clear that limiting the rise time is very important to lower high frequency content.

Some examples:

Tr	F2
1uS	0.318 MHz
100nS	3.18 MHz
10nS	31.8 MHz
1nS	318 MHz

Effect of Duty Cycle: As duty cycle approaches zero, max amplitude also approaches zero, meaning low duty cycle pulse trains will emit less energy. Note that this is balanced with the F_1 breakpoint moving higher as D → 0. The highest max amplitude occurs as Duty cycle → 100%.

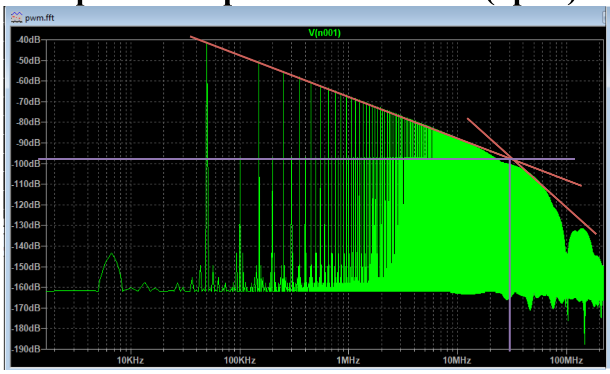


Increasing the duty cycle raises the max amplitude but lowers the breakpoint of the 20dB/decade fall off.

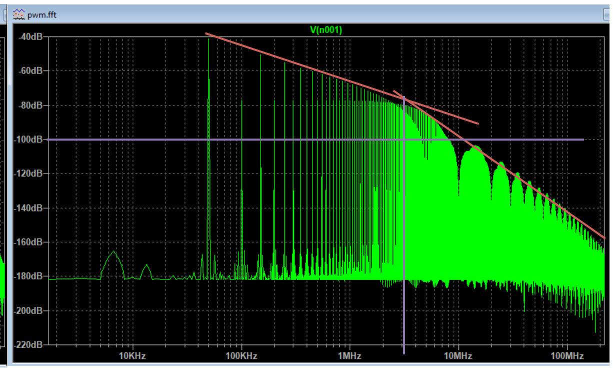
Examples (for a 1V, 1MHz pulse train)

D	Ampl	F1
1%	0.02	31.8 MHz
10%	0.2	3.18 MHz
50%	1.0	0.64 MHz
90%	1.8	0.35 MHz

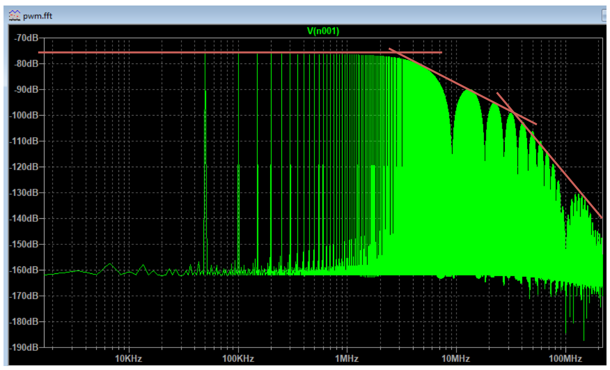
Examples with Spectral Simulation (Spice)



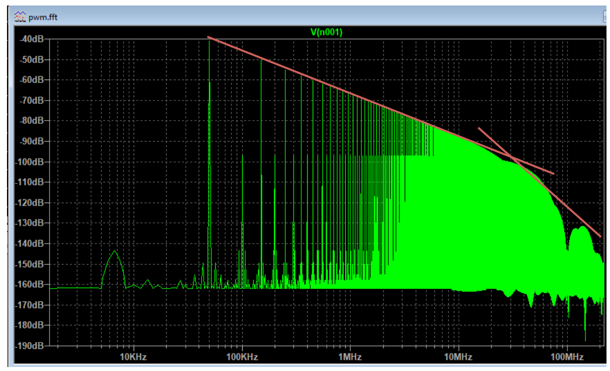
F_0 = 10 KHz, T_r = **10nS**, D = 50%



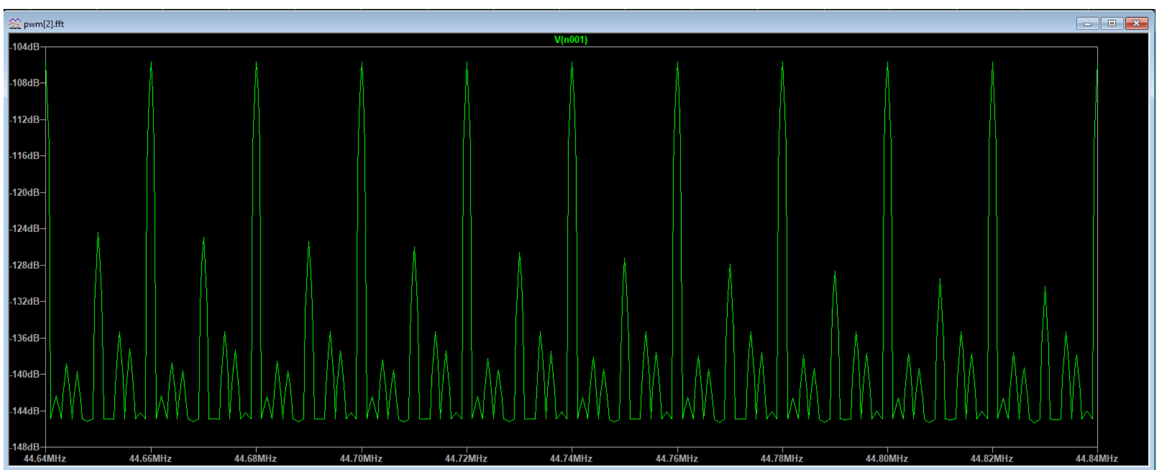
F_0 = 10 KHz, T_r = **100nS**, D = 50%



F_0 = 10 KHz, T_r = 10nS, D = **1%**



F_0 = 10 KHz, T_r = 10nS, D = **99%**



Spectrum Detail – 10 kHz waveform at 44 MHz – Note repeating harmonics at 10 KHz Intervals

References:

Electromagnetic Compatibility, Clayton Paul, Wiley
"Spectra of Digital Clock Signals", Bogdon Adamczyk, InCompliance Magazine